

Sunday, November 17, 2024

Weyl, Symmetry, and the MOG

(HTML version of an earlier post)

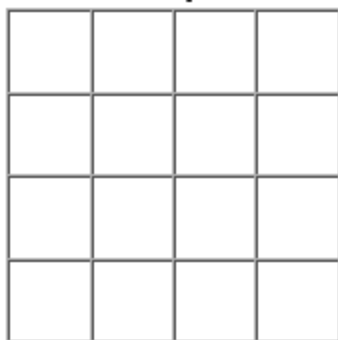
Filed under: [General](#) — Tags: [Entities](#) — m759 @ 1:16 am [Edit This](#)

Some historical background for a new book by Robert T. Curtis,
The Art of Working with the Mathieu Group M_{24} —

"Space is another example of an entity endowed with a structure. Here the elements are points, and the structure is established in terms of certain basic relations between points such as: A, B, C lie on a straight line, AB is congruent CD, and the like. What we learn from our whole discussion and what has indeed become a guiding principle in modern mathematics is this lesson: *Whenever you have to do with a structure endowed entity Σ try to determine its group of automorphisms*, the group of those element-wise transformations which leave all structural relations undisturbed. You can expect to gain a deep insight into the constitution of Σ in this way. After that you may start to investigate symmetric configurations of elements, i.e. configurations which are invariant under a certain subgroup of the group of all automorphisms; and it may be advisable, before looking for such configurations, to study the subgroups themselves, e.g. the subgroup of those automorphisms which leave one element fixed, or leave two distinct elements fixed, and investigate what discontinuous or finite subgroups there exist, and so forth."

— Hermann Weyl, *Symmetry*, Princeton University Press, 1952.
(Page 144 in the Princeton Science Library edition of 1989.)

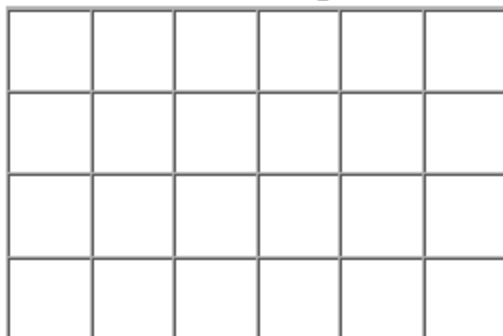
4×4 Square



This square's automorphism group
has 322,560 transformations.

— The *diamond theorem* of Steven H. Cullinane.

4×6 Rectangle



This rectangle's automorphism group
has 244,823,040 transformations.

— The *Miracle Octad Generator* (MOG) of Robert T. Curtis.

The rectangle's automorphism group contains the square's as a subgroup. The square's automorphism group leaves invariant a set of 30 eight-subsquare sets called *affine hyperplanes*. The rectangle's automorphism group leaves invariant a set of 759 eight-subsquare sets called *octads*.